Reg. No. : | | | | | | | | |

G. VENKATASWAMY NAIDU COLLEGE (AUTONOMOUS), KOVILPATTI – 628 502.

PG DEGREE END SEMESTER EXAMINATIONS - NOVEMBER 2024.

(For those admitted in June 2023 and later)

PROGRAMME AND BRANCH: M.Sc., MATHEMATICS

SEM	CATEGORY	COMPONENT	COURSE CODE	COURSE TITLE
I	PART - III	CORE-2	P23MA102	REAL ANALYSIS I

Date : 06.11.2024 / AN   Time : 3 hours   Maximum: 75 Marks

Course Outcome	Bloom's K-level	Q. No.	**SECTION – A (10 X 1 = 10 Marks)** Answer **ALL** Questions.
CO1	K1	1.	If f is monotonic on $[a, b]$, then the set of discontinuities of f is ___. a) finite b) infinite c) countable d) uncountable
CO1	K2	2.	A series $\sum a_n$ is absolutely convergent if $\sum \lvert a_n \rvert$ ________. a) converges b) diverges c) converges to 0 d) None of the above
CO2	K1	3.	$P' \supseteq P$ implies ________. a) $\lVert P' \rVert = \lVert P \rVert$ b) $\lVert P' \rVert \leq \lVert P \rVert$ c) $\lVert P' \rVert \geq \lVert P \rVert$ d) $\lVert P' \rVert \neq \lVert P \rVert$
CO2	K2	4.	Let $\alpha \nearrow [a, b]$. The upper Stieltjes integral of α and the lower Stieltjes integral of α are defined as $\bar{I}(f, \alpha) = \inf\{U(P, f, \alpha) : P \in \mathcal{P}[a, b]\}$ $\underline{I}(f, \alpha) = \sup\{L(P, f, \alpha) : P \in \mathcal{P}[a, b]\}$. Give the relation between these two integrals. a) $\bar{I}(f, \alpha) = \underline{I}(f, \alpha)$ b) $\bar{I}(f, \alpha) \geq \underline{I}(f, \alpha)$ c) $\bar{I}(f, \alpha) \leq \underline{I}(f, \alpha)$ d) $\bar{I}(f, \alpha) \neq \underline{I}(f, \alpha)$
CO3	K1	5.	The sufficient condition for the existence of the Riemann integral $\int_a^b f(x)dx$ is ________. a) f is continuous on $[a, b]$ b) f is of bounded variation on $[a, b]$ c) both a) and b) d) None of the above

CO3	K2	6.	Let f be defined and bounded on an interval S. If $T \subseteq S$, the number oscillation of f on T is _________. a) $inf\{f(x) - f(y): x \in T, y \in T\}$ b) $sup\{f(x) - f(y): x \in T, y \in T\}$ c) $inf\{f(x) - f(y): x \in S, y \in T\}$ d) $sup\{f(x) - f(y): x \in S, y \in T\}$
CO4	K1	7.	Let $f(p,q) = \frac{pq}{p^2+q^2}$. Then $\lim_{q\to\infty} f(p,q) =$ ______. a) p b) $\frac{1}{p}$ c) 1 d) 0
CO4	K2	8.	Let $a_n \geq 0$. Then the product $\prod(1 + a_n)$ converges if and only if $\sum a_n$ ________. a) converges b) diverges c) conditionally converges d) none of the above
CO5	K1	9.	Let $f_n(x) = \frac{\sin nx}{\sqrt{n}}$ if $x \in R, n = 1,2,\dots$. Then $\lim_{n\to\infty} f_n(x) =$ ____________ for every x. a) 1 b) 1/2 c) 2 d) 0
CO5	K2	10.	Let $f_n(x) = x^n$ if $0 \leq x \leq 1$. Then $\lim_{n\to\infty} \int_0^1 f_n(x) =$ _______. a) 0 b) e c) 2 d) 1
Course Outcome	**Bloom's K-level**	**Q. No.**	**SECTION – B (5 X 5 = 25 Marks)** **Answer ALL Questions choosing either (a) or (b)**
CO1	K2	11a.	i. If f is monotonic on $[a,b]$, then prove that f is of bounded variation on $[a,b]$. ii. If f is continuous on $[a,b]$ and if f' exists and is bounded in the interior, say $\lvert f'(x)\rvert \leq A$ for all x in (a,b), then show that f is of bounded variation on $[a,b]$. **(OR)**
CO1	K2	11b.	Let $\sum a_n$ be a given series with real-valued terms and define $p_n = \frac{\lvert a_n\rvert + a_n}{2}$, $q_n = \frac{\lvert a_n\rvert - a_n}{2}$ (n = 1, 2, …) Then show that if $\sum a_n$ is conditionally convergent, both $\sum p_n$ and $\sum q_n$ diverge.
CO2	K2	12a.	State and Prove Euler's Summation Formula. **(OR)**
CO2	K2	12b.	Assume that $\alpha \nearrow$ on $[a,b]$. If $f \in R(\alpha)$ and $g \in R(\alpha)$ on $[a,b]$ and if $f(x) \leq g(x)$ for all x in $[a,b]$, then show that $\int_a^b f(x)\, d\alpha(x) \leq \int_a^b g(x)\, d\alpha(x)$

CO3	K3	13a.	If f is continuous on $[a,b]$ and if α is of bounded variation on $[a,b]$, then prove that $f \epsilon R(\alpha)$ on $[a,b]$.
			(OR)
CO3	K3	13b.	Assume that α is continuous and that $f \nearrow$ on [a, b]. Then prove that there exists a point x_0 in [a, b] such that $$\int_a^b f(x)\, d\alpha(x) = f(a) \int_a^{x_0} d\alpha(x) + f(b) \int_{x_0}^b d\alpha(x)$$
CO4	K3	14a.	If a series is convergent with sum S, then prove that it is also (C, 1) summable with Cesaro sum S.
			(OR)
CO4	K3	14b.	Write the statement of Tauber's Theorem and also prove that this theorem.
CO5	K4	15a.	Assume that $f_n \to f$ uniformly on S. If each f_n is continuous at a point c of S, then prove that the limit function f is also continuous at c.
			(OR)
CO5	K4	15b.	Assume that $\lim_{n\to\infty} f_n = f$ on $[a,b]$. If $g \in R$ on $[a,b]$, define $h(x) = \int_a^x f(t)g(t)dt,\ h_n(x) = \int_a^x f_n(t)g(t)dt,$ if $x \in [a,b]$. Then prove that $h_n \to h$ uniformly on $[a,b]$.

Course Outcome	Bloom's K-level	Q. No	**<u>SECTION – C</u> (5 X 8 = 40 Marks)** **Answer <u>ALL</u> Questions choosing either (a) or (b)**
CO1	K4	16a.	Assume that f is of bounded variation on $[a,b]$, and assume that $c \in [a,b]$. Then prove that f is of bounded variation on $[a,c]$ and on [c, b] and also prove that $V_f(a,b) = V_f(a,c) + V_f(c,b)$.
			(OR)
CO1	K4	16b.	Assume that f is of bounded variation on $[a,b]$. If $x \in (a,b]$, let $V(x) = V_f(a,x)$ and put $V(a) = 0$. Then prove that every point of continuity of f is also a point of continuity of V. Also prove that the converse is true.
CO2	K5	17a.	Assume $f \epsilon R(\alpha)$ on $[a,b]$ and assume that α has a continuous derivative α' on $[a,b]$. Then prove that the Riemann Integral $\int_a^b f(x)\alpha'(x)dx$ exists and also prove that $\int_a^b f(x)\, d\alpha(x) = \int_a^b f(x)\alpha'(x)dx$.
			(OR)
CO2	K5	17b.	Assume that $\alpha \nearrow$ on $[a,b]$. Then prove that the following three statements are equivalent. i. $f \epsilon R(\alpha)$ on $[a,b]$.

			ii. f satisfies Riemann's condition with respect to α on $[a,b]$. iii. $\underline{I}(f,\alpha) = \overline{I}(f,\alpha)$.
CO3	K5	18a.	Assume that α is of bounded variation on $[a,b]$. Let $V(x)$ denote the total variation of α on $[a,x]$ if a < x $\leq b$, and let $V(a) = 0$. Let f be defined and bounded on $[a,b]$. If $f \in R(\alpha)$ on $[a,b]$, then prove that $f \in R(V)$ on $[a,b]$. **(OR)**
CO3	K5	18b.	State and Prove Lebesgue's criterion for Riemann-Integrability.
CO4	K5	19a.	Assume that each $a_n \geq 0$. Then prove that the product $\prod(1 - a_n)$ converges if and only if the series $\sum a_n$ converges. **(OR)**
CO4	K5	19b.	State and prove Abel's Limit Theorem.
CO5	K6	20a.	Let α be of bounded variation on $[a,b]$. Assume that each term of the sequence $\{f_n\}$ is a real-valued function such that $f_n \in R(\alpha)$ on $[a,b]$ for each n = 1, 2, ... Assume that $f_n \to f$ uniformly on $[a,b]$and define $g_n(x) = \int_a^x f_n(t)\, d\alpha(t)$ if $x \in [a,b]$, n = 1,2, ... Then prove that i. $f \in R(\alpha)$ on $[a,b]$. ii. $g_n \to g$ uniformly on [a, b], where $g(x) = \int_a^x f(t)\, d\alpha(t)$. **(OR)**
CO5	K6	20b.	State and prove Dirichlet's test for uniform convergence.